Rules of Probability

- Definition of conditional probability: P(A | B) = P(A, B) / P(B)
- Product rule: P(A, B) = P(A | B)P(B) = P(B | A)P(A)
  - Bayes' rule: P(A | B) = P(B | A)P(A) / P(B)
- Chain rule: P(A, B, C) = P(A | B, C)P(B, C) = P(A | B, C)P(B | C)P(C)
- P(A or B) = P(A v B) = P(A) + P(B) – P(A, B)
- P(~A) = 1 – P(A)
- Marginalization, or summing out: $P(Y) = \sum_z P(Y, Z = z) = \sum_z P(Y, z)$
- Conditioning, or law of total probability: $P(Y) = \sum_z P(Y, z) = \sum_z P(Y| z)P(z)$
  - Example for binary rvs: P(a) = P(a, b) + P(a, ~b) = P(a | b)P(b) + P(a | ~b)P(~b)
- General inference in a full joint probability distribution: Given we want to find a probability distribution for rv X, given the values (e) of evidence variables E, and Y represents the remaining unknown variables.

$$P(\boldsymbol{X} \mid e) = \alpha \cdot P(\boldsymbol{X}, e) = \alpha \sum_{y \in Y} P(\boldsymbol{X}, e, y)$$

- If X and Y are (marginally) independent, then
  P(X, Y) = P(X)P(Y) and P(X | Y) = P(X) and P(Y | X) = P(Y)
- If X and Y are conditionally independent given Z, then
  P(X, Y | Z) = P(X | Z)P(Y | Z) and P(X | Y, Z) = P(X | Z) and P(Y | X, Z) = P(Y | Z)

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576