AI Homework 3 – Probability

1. Suppose Professor Phillips is visiting an amusement park where the following events might happen: riding a roller coaster (R), eating a soft pretzel (P), feeling sick (probably because of eating pretzels or riding the roller coaster) (S), and feeling hungry (H).

   You are given the following full joint probability distribution over these four events, each of which either happens or doesn't happen.

Roller Coaster (R)	Eat Pretzel (P)	Feel Sick (S)	Feel Hungry (H)	Joint probability
T	T	T	T	0.144
T	T	T	F	0.216
T	T	F	T	0.016
T	T	F	F	0.024
T	F	T	T	0.096
T	F	T	F	0.024
T	F	F	T	0.224
T	F	F	F	0.056
F	T	T	T	0.008
F	T	T	F	0.012
F	T	F	T	0.032
F	T	F	F	0.048
F	F	T	T	0.008
F	F	T	F	0.002
F	F	F	T	0.072
F	F	F	F	0.018

   Use the table to calculate the following probabilities. Show all your work.

   (a) P(S, P, R)
   (b) P(¬S | R, P)
   (c) P(R, P | ¬S)
   (d) P(R | P, ¬H)

2. James Bond likes to drink vodka martinis, and he prefers them "shaken, not stirred." (Every vodka martini must be either "shaken" or "stirred," but not both at once, like heads/tails on a coin.)

   Suppose Bond knows that
   the probability that a vodka martini tastes good given that it has been shaken is 0.9, whereas
   the probability that a vodka martini tastes good given that it has been stirred is 0.5.

   Bond orders a vodka martini but forgets to specify how he prefers it. When the martini arrives, Bond asks the server how it was prepared. The server doesn't remember, but estimates the probability of it being shaken is 0.4, while the probability it is stirred is 0.6. Bond is upset that the server isn't more specific, but drinks the martini anyway and notices that it tastes good.

What is the probability that Bond's martini was shaken, given that it tasted good? Show your work in calculating this number.

Hint: Do not use two events for shaken & stirred; these are opposites in this situation, so pick one of them, and let the other one be the "not" version of the first event.

3. Punxsutawney Phil is a groundhog that lives in Pennsylvania, who supposedly can predict how long winter will last. On Groundhog Day, February 2, Phil emerges from his burrow and (according to tradition) if he sees his shadow, there will be six more weeks of winter. If he doesn't see his shadow, there will be an early spring.

One year, a new groundhog, Punxsutawney Jill, comes to town, and she claims that she makes better predictions than Punxsutawney Phil. Here are their statistics:

When an early spring is going to happen, Phil detects it correctly 90% of the time. (In other words, *given that* it's an early spring, Phil predicts correctly 90% of the time.) However, he has a 15% false positive rate (when he predicts an early spring *given that* it doesn't actually happen). On the other hand, Jill correctly recognizes true early springs only 85% of the time, but she only has a 10% false positive rate. Furthermore, early springs only happen 20% of the time.

Answer the following questions. Show your work. Hint: Use three events here: E=there is an early spring. P=Phil predicts the weather correctly. J=Jill predicts the weather correctly. **I suggest beginning by just translating the description above into marginal and/or conditional probabilities using E, P, and J, and writing them down.**

(a) If Phil predicts an early spring, what is the probability that the prediction is correct? In other words, *given that* Phil predicts an early spring, what is the probability that the early spring occurs?

(b) If Jill predicts an early spring, what is the probability that the prediction is correct?

(c) What is Phil's overall prediction accuracy? (What is the probability he makes a correct prediction, regardless of the weather?) Hint: this is a marginal (not conditional) probability.

(d) What is Jill's overall prediction accuracy?