

# Recursive Functions with Arrays

# Iterative version

- Have an array/ArrayList called array. Want to find the sum of all the elements:

```
int sum = 0;
for (int i = 0; i < array.size(); i++)
    sum += array[i];
```

# Recursive version

- Base case: What is the smallest size of an array for which we would ever want to add up all the elements?
- Recursive case:
  - Suppose you have an array with  $>1$  element.
  - How can I describe finding the sum of all the elements as involving *finding the sum of the elements of a smaller sized array?*
  - Hint: Suppose my array has 5 elements. My best friend knows how to find the largest value in an array, but only for 4 elements. How can I use him to solve my problem?

# Recursive version

- `sum(array)`
- Base case: If `array.size() == 1`, return `array[0]`
- Recursive case: If `array.size() > 1`:
  - Compute the sum of all the elements in the sub-array from index 1 to the end → `sum(array[1:])` in Python syntax.
  - Add `array[0]` to this sum from above.
  - Return this value.

array = [7, 9, 8]

- `sum(array)`
- Base case: If `array.size() == 1`,  
return `array[0]`
- Recursive case: If `array.size() > 1`:
  - Find the sum of elements in  
`array[1:]` (whole array except `A[0]`)
    - call it `smallerSum`
  - Return `smallerSum + array[0]`

Call `sum([7, 9, 8])`

`array = [7, 9, 8]`  
`smallerSum = sum([9, 8])`

Call `sum([9, 8])`

`array = [9, 8]`  
`smallerSum = sum([8])`

Call `sum([8])`

`array = [8]`  
Base case!

array = [7, 9, 8]

- `sum(array)`
- Base case: If `array.size() == 1`, return `array[0]`
- Recursive case: If `array.size() > 1`:
  - Find the sum of elements in `array[1:]` (whole array except `A[0]`)
    - call it `smallerSum`
  - Return `smallerSum + array[0]`

Call `sum([7, 9, 8])`

array = [7, 9, 8]  
smallerSum = `sum([9, 8])`

Call `sum([9, 8])`

array = [9, 8]  
smallerSum = `sum([8]) = 8`

Call `sum([8])`

array = [8]  
Base case!  
Return `array[0] = 8`

Returns  
8



array = [7, 9, 8]

- `sum(array)`
- Base case: If `array.size() == 1`, return `array[0]`
- Recursive case: If `array.size() > 1`:
  - Find the sum of elements in `array[1:]` (whole array except `A[0]`)
    - call it `smallerSum`
  - Return `smallerSum + array[0]`

Call `sum([7, 9, 8])`

array = [7, 9, 8]  
smallerSum = `sum([9, 8])`

Call `sum([9, 8])`

array = [9, 8]  
smallerSum = `sum([8]) = 8`  
Return `smallerSum + array[0]`

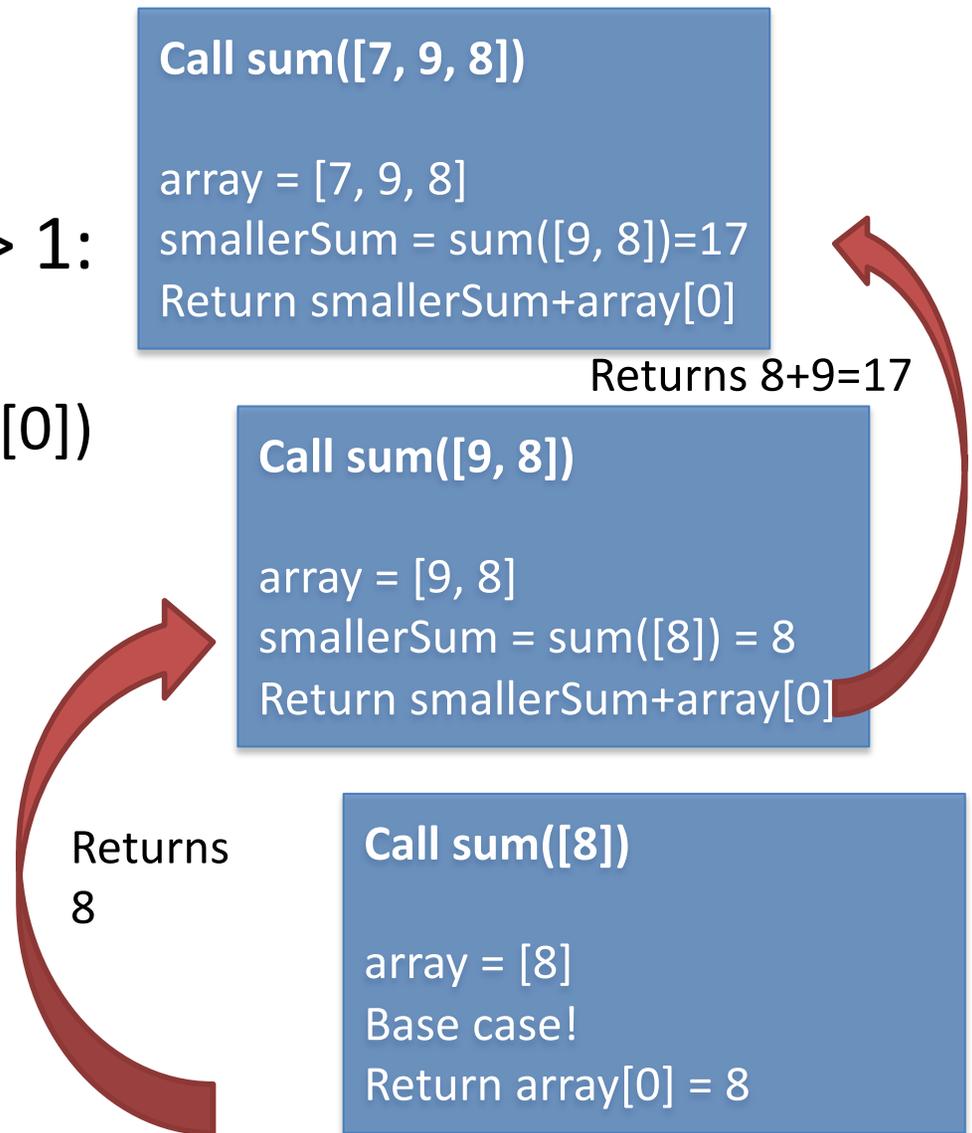
Call `sum([8])`

array = [8]  
Base case!  
Return `array[0] = 8`

Returns  
8

array = [7, 9, 8]

- `sum(array)`
- Base case: If `array.size() == 1`, return `array[0]`
- Recursive case: If `array.size() > 1`:
  - Find the sum of elements in `array[1:]` (whole array except `A[0]`)
    - call it `smallerSum`
  - Return `smallerSum + array[0]`



array = [7, 9, 8]

- sum(array)
- Base case: If array.size() == 1  
return array[0]
- Recursive case: If array.size() > 1:
  - Find the sum of elements in array[1:] (whole array except A[0])
    - call it smallerSum
  - Return smallerSum + array[0]

Returns 17+7=24

Call sum([7, 9, 8])

array = [7, 9, 8]

smallerSum = sum([9, 8])=17

Return smallerSum+array[0]

Returns 8+9=17

Call sum([9, 8])

array = [9, 8]

smallerSum = sum([8]) = 8

Return smallerSum+array[0]

Returns  
8

Call sum([8])

array = [8]

Base case!

Return array[0] = 8

# Java recursive version

- Java does let you take slices of arrays like Python, it involves using some techniques we haven't learned yet, so we're going to see a different way.
- Notice that our slices always involving chopping off the first element in the array; i.e,  $A[0]$ 
  - $[7, 9, 8] \rightarrow [9, 8] \rightarrow [8]$
- How can we simulate an array slice without actually doing the slicing?
  - Hint: In the olden days, people used these things called "bookmarks" to hold their spot in a book while they were reading. We can use the same idea to mark the section of the array that we are interested in recursing on.

# Java recursive version

- Use an integer variable "bookmark" to save your spot in the array.
- When we make a recursive call, instead of passing an updated array (like the Python version), we will pass an updated bookmark.
- Our function will now be `sum(array, leftIdx)`
  - `leftIdx` ("left index") represents the index of the bookmark in the array: everything before the bookmark is already read, everything afterwards is unread. So it is the leftmost index of the portion of the array we have still left to read.

# Recursive Java version

- `sum(A, leftIdx)`
- Base case: ???
- Recursive case:
  - Find the sum of elements in ???
    - call it `smallerSum`
  - return ???
- Where does the bookmark start?

# Recursive Java version

- `sum(A, leftIdx)`
- Base case: if `leftIdx == array.size() - 1`
- Recursive case:
  - Find the sum of elements in everything after `array[leftIdx]`
    - `smallerSum = sum(array, leftIdx + 1)`
  - return `smallerSum + array[0]`
- Initial call should be `sum(array, 0)`

- How can we use this idea to find the largest element in an array/arraylist? (the max element)?
- On paper, write out a recursive formulation for this.
  - What is the base case? (What is the smallest size of an array we would want to take the max of?)
  - What is the recursive case? For a bigger array, how do we find the max element by reducing the problem to a smaller version of the same problem?
  - Hint: For an array of size  $\geq 2$ , suppose a friend tells you they have already computed the largest element in the sub-array from index 1 to the end. How can you use this information to help you find the overall largest element?
- Code this in Java!